



Oct 14-9:32 AM

4.4c Modeling and Optimization

Examples from Economics

Maximum Profit: If there is a maximum profit, it occurs when marginal revenue = marginal cost

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$P = R - C$$

$$P' = R' - C' = 0$$

$$R' = C'$$

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Suppose $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents 1000's of units. Is there a production level that maximizes profit? If so, what is it?

$$R' = C' \quad \text{X}$$

$$P = R - C$$

$$P' = R' - C'$$

$$9 = 3x^2 - 12x + 15$$

$$0 = 3x^2 - 12x + 6$$

$$x = \sqrt{2} + 2 \approx 3.414$$

$$P' = 9 - (3x^2 - 12x + 15)$$

$$P'' = -6x + 12 \Big|_{x=3.414} < 0$$



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average cost = $c(x)/x$

Minimum Average Cost: If there is a minimum average cost, it occurs when average cost = marginal cost.

$$A = \frac{c(x)}{x}$$

$$A' = \frac{x \cdot c'(x) - c(x) \cdot 1}{x^2} = 0$$

$$x c'(x) - c(x) = 0$$

$$x \left[\frac{c'(x) - \frac{c(x)}{x}}{1} \right] = 0$$

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Suppose $c(x) = x^3 - 6x^2 + 15x$, where x represents 1000's of units. Is there a production level that minimizes average cost? If so, what is it?

$$\begin{aligned}
 A &= \frac{c(x)}{x} = c'(x) & A &= \frac{c(x)}{x} \\
 \frac{x^3 - 6x^2 + 15x}{x} &= 3x^2 - 12x + 15 \\
 A &= x^2 - 6x + 15 = 3x^2 - 12x + 15 \\
 A' &= 2x - 6 & 0 &= 2x^2 - 6x = 2x(x-3) \\
 A'' &= 2 > 0 \quad \text{😊} & X &= 3
 \end{aligned}$$

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